eters all zero. The desired response $x^*(t)$ is recorded in channel 1, and the output $x(t)$ with additive noise which collectively represents the data $z(t)$ is recorded in channel 2. The estimates $\hat{a}_1(t)$, $\hat{a}_2(t)$, and $\hat{a}_3(t)$ are recorded in channels 3, 4, and 5. The estimation of $\hat{a}_1(t)$ takes considerably longer than that of $\hat{a}_2(t)$ and $\hat{a}_3(t)$, because, once the steady-state response to the step input (square wave input) is reached, no further information about the relative damping can be obtained. In Fig. 9 the desired response has been changed to 0.35 critical damping $[a_1^*(t) = 0.7, a_2^*(t) = -1, a_3^*(t) = +1]$ with $a_1(t)$, $a_2(t)$, and $a_3(t)$ equal to -1.4, -1, and $+1$. Initial estimates $a_1(t)$, $a_2(t)$, and $a_3(t)$ are all zero. Figure 10 shows the condition when $a_3(t)$ varies linearly with time, with all initial estimates zero and a random input. It should be noted that the system can recognize the cope with control reversal.

The particular runs shown here represent only a few of the many runs made and were chosen to be representative of different conditions under which such a system might operate. A first-order system, with variable time constant and gain that was actually studied prior to the second-order system illustrated throughout this paper, was also simulated, with equally good results.

Conclusions

The primary purpose of this paper was to develop a statistical means of identifying the parameters of a linear system from noisy data and to show how the method might be applied to an adaptive control system. Although many approximations and assumptions were made, excellent results have been demonstrated by experiment.

If additional measurement data are available (x_2 in example discussed), $\mathbf{M}$ and Ψ become rectangular matrices, v becomes a column vector, and the variance σ_v^2 becomes a covariance matrix. The vector matrix equations remain the same.

The reader might feel that this particular demonstration is indeed trivial compared to a more general formulation that has been postulated. Certainly this is a valid concern, to which the following comment is offered. Observe that Eq. (1) is a nonlinear equation in the variables x_1, x_2, a_1, a_2, and a_3. One seeks to estimate these variables given data $\mathbf{z}$, albeit postulating certain not unreasonable behavior of the elements a_1, a_2, and a_3. From this point of view, the reader is referred to the works of Battin[6] and others[7] which successfully use a similar technique of sequential linear regression to estimate the state of a six-dimensional nonlinear system (space vehicle trajectory determination). Actually, it was that application of Kalman's techniques which led the authors to consider the identification problem from the point of view expressed in this paper. The excellent results obtained by Battin et al. would seem to justify the authors' expectation of successful application of the proposed identification scheme to higher-order systems.

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OCTOBER 1963 AIAA JOURNAL VOL. 1, NO. 10

Rise and Set Time of a Satellite about an Oblate Planet

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A technique is presented for the determination of the rise and set times of a satellite about an oblate planet. The solution depends on the ability to determine the rise and set eccentric anomalies from a single transcendental equation. The solution of the transcendental equation is facilitated greatly by an approximating technique. The cases of both zero and minimum elevation angles are analyzed. Numerical results with a strictly integrated orbit are presented for comparison with the closed-form solution.

THE purpose of this paper is to present a Keplerian closed-form solution to the rise and set time problem. In effect, this problem usually involves the calculation of the rise and set universal time of a given satellite from a specific ground station.

In the past, it has been the custom to solve the problem by letting the satellite run through its ephemeris and checking at each instant to see whether the elevation angle h of the satellite with respect to a ground station was greater than some minimum value. However, by attacking the problem from a different point of view, i.e., with the eccentric anomaly taken to be the independent variable, it is possible to obtain a closed-form solution to the satellite visibility problem. Specifically, the closed-form solution is a single transcendental equation in the eccentric anomalies corresponding to a rise and set time for a given orbital pass of a satellite. It is more difficult to solve the controlling equation than the standard Keplerian equation. However, the method offers the advantage that the controlling equation is solved only once per orbital period, as contrasted with the hundreds of times the Keplerian equation must be solved by the standard step-by-step technique.

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If the orbital elements and the station coordinates as well as the minimum value of station elevation angle h are known, it will be possible to obtain a single transcendental equation in the eccentric anomalies corresponding to the rise and set times.

The orbital elements a, e, i, Ω, ω, and T are defined as follows:

a = semimajor axis
e = eccentricity
i = equatorial inclination
Ω = longitude of ascending node
ω = argument of perigee
T = time of perifocal passage

The station coordinates θ_0, φ, λ_E, and H are defined as follows:

θ_0 = station sidereal time at some epoch time
φ = station geodetic latitude
λ_E = station east longitude
H = station height above and measured normal to the surface of the adopted ellipsoid

This paper uses the same terminology as that employed by Baker and Makemson[1] and by Herrick.[2] The symbols used here also are identical to Baker and Makemson's, except for those specifically redefined in this text.

Development of the Controlling Equation Which Defines the Rise and Set Eccentric Anomalies

From a ground station located in the rotating topocentric coordinate frame (Fig. 1), the sine of the elevation angle is given by

$$\boldsymbol{\rho}\cdot \mathbf{Z}/\rho = \sin h \tag{1}$$

or

$$\rho_x Z_x + \rho_y Z_y + \rho_z Z_z = \rho \sin h \tag{2}$$

where the components of the unit vector $\mathbf{Z}$ point toward the geodetic zenith. It should be noted that $\mathbf{Z}$ is defined usually as pointing towards the astronomical zenith; however, for the solution of the rise and set problem, it is much more convenient to work in the geodetic system. Referred to an inertial coordinate frame (Fig. 2), the components of $\mathbf{Z}$ are given directly by

$$Z_x = \cos\theta \cos\varphi \qquad Z_y = \sin\theta \cos\varphi \qquad Z_z = \sin\varphi \tag{3}$$

where θ is the station sidereal time, and φ is the station geodetic latitude.

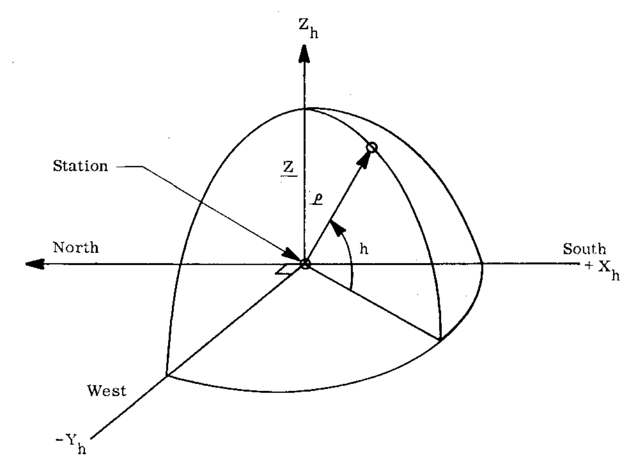

Fig. 1 Satellite location in a topocentric coordinate system.

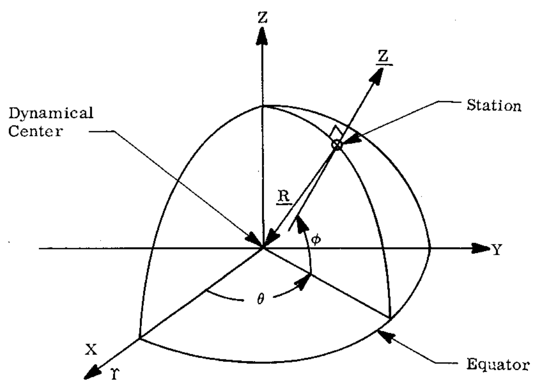

Fig. 2 Station coordinate system in a geocentric coordinate system.

The relationship between the observation station, satellite, and dynamical center (Fig. 3) is defined by

$$\boldsymbol{\rho} = \mathbf{r} + \mathbf{R} \tag{4}$$

where $\boldsymbol{\rho}$ is the slant range vector, $\mathbf{r}$ is the orbit radius vector, and $\mathbf{R}$ is the station coordinate vector.

Hence, in terms of the geodetic latitude, the components of the station radius vector $\mathbf{R}$ for an oblate spheroid are given by

$$\begin{aligned} X &= -G_1 \cos\varphi \cos\theta \\ Y &= -G_1 \cos\varphi \sin\theta \\ Z &= -G_2 \sin\varphi \end{aligned} \tag{5}$$

where

$$G_1 = \{a_e/[1 - (2f - f^2)\sin^2\varphi]^{1/2}\} + H$$

$$G_2 = \{(1 - f)^2 a_e/[1 - (2f - f^2)\sin^2\varphi]^{1/2}\} + H$$

and

a_e = equatorial radius of the planet
f = flattening of the adopted ellipsoid
H = station elevation above and measured normal to the surface of the adopted ellipsoid
θ = local sidereal time

Having defined the necessary variables and coordinate systems, it is now possible to substitute Eq. (4) into Eq. (2) so that

$$(x + X)Z_x + (y + Y)Z_y + (z + Z)Z_z = \rho \sin h \tag{6}$$

Introducing the unit vector $\mathbf{Z}$ and the station coordinates into Eq. (6), it is possible to write

$$(x - G_1 \cos\varphi \cos\theta) \cos\varphi \cos\theta + (y - G_1 \cos\varphi \sin\theta) \cos\varphi \sin\theta + (z - G_2 \sin\varphi) \sin\varphi = \rho \sin h \tag{7}$$

or, upon rearrangement,

$$x \cos\varphi \cos\theta + y \cos\varphi \sin\theta + z \sin\varphi = \rho \sin h + G \tag{8}$$

where

$$G = G_1 \cos^2\varphi + G_2 \sin^2\varphi$$

Equation (8) is, in effect, the geometric constraint that comprises the controlling equation.

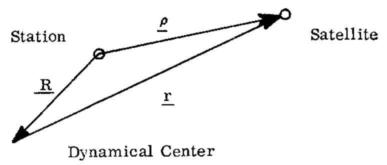

Fig. 3 Relationship of station, satellite, and dynamical center.

Analytic Solution of the Controlling Equation When the Satellite Is on the Geodetic Horizon

When the elevation angle at the ground station is taken to be zero, the controlling equation can be written as

$$x \cos\varphi \cos\theta + y \cos\varphi \sin\theta + z \sin\varphi = G \tag{9}$$

Introducing the vector equation

$$\mathbf{r} = x_\omega \mathbf{P} + y_\omega \mathbf{Q} \tag{10}$$

where **P** and **Q** unit vectors are given by

$$\begin{aligned} P_x &= \cos\omega \cos\Omega - \sin\omega \sin\Omega \cos i \\ P_y &= \cos\omega \sin\Omega + \sin\omega \cos\Omega \cos i \\ P_z &= \sin\omega \sin i \\ Q_x &= -\sin\omega \cos\Omega - \cos\omega \sin\Omega \cos i \\ Q_y &= -\sin\omega \sin\Omega + \cos\omega \cos\Omega \cos i \\ Q_z &= \cos\omega \sin i \end{aligned} \tag{11}$$

transforms Eq. (9) into an equation in which z is lacking, i.e., in which analysis is now being performed in the orbital plane. Therefore, the controlling equation can be expressed as follows:

$$\begin{aligned}&[P_x \cos\varphi \cos\theta + P_y \cos\varphi \sin\theta + P_z \sin\varphi]x_\omega + \\ &\quad [Q_x \cos\varphi \cos\theta + Q_y \cos\varphi \sin\theta + Q_z \sin\varphi]y_\omega = G\end{aligned} \tag{12}$$

or more concisely, as

$$\mathbf{P}\cdot\mathbf{Z}x_\omega + \mathbf{Q}\cdot\mathbf{Z}y_\omega = G \tag{13}$$

Since the coordinates in the orbit plane are given by

$$x_\omega = a(\cos E - e) \qquad y_\omega = a[1 - e^2]^{1/2} \sin E \tag{14}$$

substitution converts Eq. (13) to the form

$$F \triangleq a(\cos E - e)\mathbf{P}\cdot\mathbf{Z} + a[1 - e^2]^{1/2} \sin E\, \mathbf{Q}\cdot\mathbf{Z} - G = 0 \tag{15}$$

Note that the dot products $\mathbf{P}\cdot\mathbf{Z}$ and $\mathbf{Q}\cdot\mathbf{Z}$ are functions of time in Eq. (15); and therefore, to write F as a function of E alone, it is necessary to eliminate the time dependency of the unit **Z** vector.

Equation (3) can be written as follows:

$$\begin{aligned} Z_x &= \cos\varphi \cos(\theta_0 + [\dot\theta(t - t_0)]) \\ Z_y &= \cos\varphi \sin(\theta_0 + [\dot\theta(t - t_0)]) \\ Z_z &= \sin\varphi \end{aligned} \tag{16}$$

where

θ_0 = epoch sidereal station time in radians
$\dot\theta$ = sidereal rate of change = constant
t_0 = epoch universal time
t = universal time

Introducing Kepler's equation, i.e.,

$$t = [(E - e \sin E)/n] + T \tag{17}$$

where n is the mean motion $= k_e(\mu)^{1/2}a^{-3/2}$, T is the time of latest perifocal passage, and $k_e(\mu)^{1/2}$ is the dynamical constant, then

$$\begin{aligned} Z_x &= \cos\varphi \cos\left(\theta_0 + \dot\theta\left[\frac{E - e\sin E}{n} + T - t_0\right]\right) \\ Z_y &= \cos\varphi \sin\left(\theta_0 + \dot\theta\left[\frac{E - e\sin E}{n} + T - t_0\right]\right) \\ Z_z &= \sin\varphi \end{aligned} \tag{18}$$

in which time has been eliminated. Therefore, Eq. (15) becomes only a function of eccentric anomaly when **Z** is defined by means of Eq. (18). Examination of Eq. (15), i.e., the controlling function, as a function of E produces a curve that has two real roots (Fig. 4) if the satellite is visible. One of the roots corresponds to the eccentric anomaly at rise, whereas the remaining one defines the set eccentric anomaly.

It can be shown that when the satellite is visible, $F > 0$, so that when F changes sign from negative to positive the satellite is rising. A set is oppositely characterized by F's changing sign from positive to negative. This in effect can be seen by consideration of the controlling function F in its simplest form, i.e.,

$$F \triangleq \boldsymbol{\rho}\cdot\mathbf{Z}/\rho - \sin h$$

Hence, by replacing $\boldsymbol{\rho}/\rho$ by its unit vector counterpart **L**, it is possible to write

$$F \triangleq \mathbf{L}\cdot\mathbf{Z} - \sin h$$

or

$$F \triangleq \cos(\angle \mathbf{L}, \mathbf{Z}) - \sin h$$

However, when the satellite is directly overhead, $\angle \mathbf{L}, \mathbf{Z} = 0$ and $F = 1 - \sin h > 0$, if $h < 90°$.

Preliminary Estimates of the Rise and Set Eccentric Anomalies

The solution of the controlling equation $F(E)$ can be facilitated greatly if some good preliminary estimates for the values of the rise and set eccentric anomalies are known. Examination of the controlling function F, at $E = 0$ and at $E = 2\pi$, produces the following two equations:

$$F(0) = \mathbf{P}\cdot\mathbf{Z}_0\, a - \mathbf{P}\cdot\mathbf{Z}_0\, ae - G = 0 \tag{19}$$

$$F(2\pi) = \mathbf{P}\cdot\mathbf{Z}_{2\pi}\, a - \mathbf{P}\cdot\mathbf{Z}_{2\pi}\, ae - G = 0 \tag{20}$$

in which it is to be noted that if the **Z** vector were constant, the controlling function F would be periodic. Therefore, the effect of **Z** is to vary the amplitude of the F function. For near-earth satellites this effect is quite small. It is therefore possible to define an average **Z** vector $\bar{\mathbf{Z}}$ by†

$$\bar{\mathbf{Z}} = (\mathbf{Z}_0 + \mathbf{Z}_{2\pi})/2 \tag{21}$$

Equation (15) can then be written as

$$\mathbf{P}\cdot\bar{\mathbf{Z}}a \cos E + \mathbf{Q}\cdot\bar{\mathbf{Z}}a[1 - e^2]^{1/2} \sin E = G + \mathbf{P}\cdot\bar{\mathbf{Z}}ae \tag{22}$$

which, upon defining the angle α by

$$\begin{aligned} \cos\alpha &= \frac{\mathbf{P}\cdot\bar{\mathbf{Z}}}{[(\mathbf{P}\cdot\bar{\mathbf{Z}})^2 + (\mathbf{Q}\cdot\bar{\mathbf{Z}})^2(1 - e^2)]^{1/2}} \\ \sin\alpha &= \frac{\mathbf{Q}\cdot\bar{\mathbf{Z}}[1 - e^2]^{1/2}}{[(\mathbf{P}\cdot\bar{\mathbf{Z}})^2 + (\mathbf{Q}\cdot\bar{\mathbf{Z}})^2(1 - e^2)]^{1/2}} \end{aligned} \tag{23}$$

reduces to

$$\cos\alpha \cos E + \sin\alpha \sin E = (1/a)(G + \mathbf{P}\cdot\bar{\mathbf{Z}}ae)\,(\cos\alpha/\mathbf{P}\cdot\bar{\mathbf{Z}}) \tag{24}$$

or

$$E = \alpha - \cos^{-1}\left[\frac{(G + \mathbf{P}\cdot\bar{\mathbf{Z}}ae)}{a[(\mathbf{P}\cdot\bar{\mathbf{Z}})^2 + (\mathbf{Q}\cdot\bar{\mathbf{Z}})^2(1 - e^2)]^{1/2}}\right] \tag{25}$$

Equation (25) is a direct expression of the value of the rise

† The averaging technique proposed by Eq. (21) will produce negative estimates [Eq. (25)] when E is either very small, i.e., nearly zero, or when very large, i.e., nearly 2π. If this should happen, it will then be advantageous to set $\bar{\mathbf{Z}}$ equal to $\mathbf{Z}(0)$ or $\bar{\mathbf{Z}}(2\pi)$, so that better estimates will result.

and set eccentric anomalies with the assumption that the planet's rotation is negligible. If the satellite is visible, this equation will have two roots. Should the argument of the arccosine be greater than unity in absolute value, it should be truncated to unity with the correct sign attached. It is possible that this will happen if the rise and set times are very close together.

For nearby satellites, numerical results have shown that Eq. (25) yields excellent results. If, however, the satellite is very far away, it will be necessary to hunt for the zeros of Eq. (15) directly. In this case a hunt with the derivative of Eq. (15), i.e., $F'(E)$, instead of $F(E)$ will be beneficial.

Numerical Solution of the Controlling Equation

Since Eq. (15) possesses two roots (if the satellite is visible) in the interval $0 \leq E \leq 2\pi$, the following procedure is recommended. Once an estimate of E is available, Newton's method provides a better or improved estimate by means of the following formula:

$$E_{n+1} = E_n - [F(E_n)/F'(E_n)] \tag{26}$$

where by differentiation of Eq. (15) [the Z's being obtained from Eq. (18)], F' is given by

$$F'(E) = \{a(\cos E - e)(P_yZ_x - P_xZ_y) + a[1 - e^2]^{1/2}\sin E(Q_yZ_x - Q_xZ_y)\}\cdot(1 - e\cos E)(\dot{\theta}/n) + \mathbf{Q}\cdot\mathbf{Z}a[1 - e^2]^{1/2}\cos E - \mathbf{P}\cdot\mathbf{Z}\cdot a\sin E \tag{27}$$

Note that the new estimate of E_{n+1} should not be allowed to change by more than about 10% of the previous value. The correction should be bounded. The Newton procedure is convergent quadratically and yields very rapid results.

Solution of the Controlling Equation When the Satellite Is above the Geodetic Horizon

Usually, a particular station imposes a minimum elevation angle constraint with respect to the rise and set times. That is, a mountain range may be in the direct line of sight. Hence, it is necessary to evaluate ρ in Eq. (8) as a function of E to be able to express the controlling equation as a function of a single parameter.

Since the slant range is given by

$$\rho = [(x + X)^2 + (y + Y)^2 + (z + Z)^2]^{1/2} \tag{28}$$

or using Eq. (5)

$$\rho = [x^2 + y^2 + z^2 + G_0^2 - (2G_1\cos\varphi\cos\theta)x - (2G_1\cos\varphi\sin\theta)y - (2G_2\sin\varphi)z]^{1/2} \tag{29}$$

it is possible to express ρ as a function of eccentric anomaly with the aid of Eqs. (3) and (14) as]

$$(\rho E) = \Big\{[a^2(1 - e\cos E)^2 + G_0^2] - 2G_1Z_x[a(\cos E - e)P_x + a\{1 - e^2\}^{1/2}\sin EQ_x] - 2G_1Z_y[a(\cos E - e)P_y + a\{1 - e^2\}^{1/2}\sin EQ_y] - 2G_2Z_z[a(\cos E - e)P_z + a\{1 - e^2\}^{1/2}\sin EQ_z]\Big\}^{1/2} \tag{30}$$

where

$$G_0 = (G_1^2\cos^2\varphi + G_2^2\sin^2\varphi)^{1/2}$$

with the understanding the $\mathbf{Z}$ is obtained from Eq. (18).

In brief, if estimates for the rise and set eccentric anomalies are known, for example, from the approximate formula Eq. (25), it is possible to compute the rise and set slant ranges from Eq. (30). Then, if the right side of Eq. (8) is redefined to be a new constant, i.e.,

$$S_k = \rho_k \sinh_k + G \qquad k = 1, 2$$

where the subscript k refers to rise and set conditions, it is possible to write Eq. (15) with S_k replacing G. Therefore, two independent equations, one for a rise eccentric anomaly and the other for a set eccentric anomaly, are obtained. The solution of Eq. (15), however, has been discussed already. The new values of E_k yield a better ρ_k, etc.

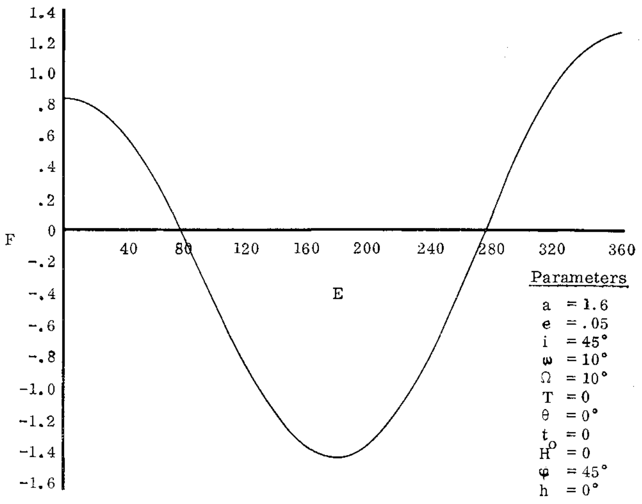

Fig. 4 *F* Function vs *E*.

Calculations of the Rise and Set Time

Once the rise and set anomalies E_k are known, where $k = 1, 2$, it is possible to solve Kepler's equation the easy way for t_k, i.e., determine the rise and set time by means of

$$t_k = [(E_k - e\sin E_k)/n] + T \qquad k = 1,2 \tag{31}$$

where T is the time of latest perifocal passage. By adding the period or multiples of it to the latest time of perifocal passage, it is possible to solve for future rise and set times

Conclusions and Results

A general set of equations have been developed for calculating the rise and set time of a satellite about a geometrically oblate planet. The analysis was programed on an IBM 7090 computer in Fortran. The secular variations in Ω and ω were added to the analysis, and the results were compared to a fully rigorous numerical integration of the equations of motion and a point-by-point check procedure. A comparison is shown in Table 1 where a is 1.0346400 earth-radii; e = 0.0004648; i = 91.995912°; Ω = 311.45905°; ω = 128.31922°; and T = 1371.0420 min.

Some of the slight discrepancies in the rise and set times can be explained as being due to the fact that the integration technique includes more extensive perturbations; the rest are due to the use of an integration method that performs a linear interpolation for the rise and set times between two integration intervals.

Table 1 Numerical comparison

Revolution number	Rise time (closed-form solution) day	hr	min	Rise time (integrated solution) day	hr	min
17	1	0	9.84	1	0	9.88
20	1	4	31.96	1	4	31.98
30	1	19	25.80	1	19	25.93
	Set time			Set time		
17	1	0	13.13	1	0	13.04
20	1	4	37.21	1	4	37.15
30	1	19	32.00	1	19	32.10

Station location (June 21, 1962)

λ_E = 291.5° h = 3.0° φ = 76.5° H = 900 ft

On the average, numerical results have shown that the computing speed for the closed-form solution is approximately 25 times faster than the computing speed for the integration procedure. Needless to say, if more perturbations are added to the analysis, the rise and set times will be computed to greater accuracy at little expense in machine time.

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OCTOBER 1963 AIAA JOURNAL VOL. 1, NO. 10

Self-Contained Satellite Navigation Systems

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Two self-contained navigation systems are described for use in a satellite travelling around a planet in an elliptical orbit. The navigation sensor for both systems is a horizon scanner that provides two pieces of data: the local vertical and the width of the planet disk. The difference between the navigation systems is the method of processing the data. One method of data processing approximates the local vertical and the disk width by polynomials in time. Using a linear filter, the polynomial fitting the data with minimum squared error is generated. Certain coefficients of this polynomial are the desired position and velocity. In the second method of data processing, the orbital parameters that fit the measurement data with minimum squared error are generated. Position and velocity are derived from these orbital parameters. Navigational accuracy is determined for both systems. Numerical results are presented to show the effect on accuracy of variations in eccentricity, major axis, accuracy of measurement data, location of satellite in orbit, and noise bandwidth. It is shown that exact fitting reduces position errors by as much as a factor of 3 and velocity errors by as much as a factor of 10 compared to polynomial fitting.

Instrumentation

BOTH orbital-rendezvous and lunar-landing missions require accurate navigation of vehicles in elliptical orbits over time intervals of 20 hr or more. For reasons of simplicity and reliability it is desirable that the navigation system be self-contained. The navigation system of interest here accomplishes this by making use of a horizon scanner in combination with a stellar-monitored inertial reference platform. The horizon scanner provides the basic measurements of the orientation of the local vertical and the width of the planet disk. The navigation quantities of interest (vehicle position and velocity) are obtained by processing and smoothing these data in an appropriate manner. The azimuth orientation of the orbital plane with respect to the planet enclosed by the orbit is determined by taking fixes on at least two stars. The subject of this paper is the method of processing the horizon-scanner data.

The desired accuracy for the navigation information in the applications under consideration is 0.5 naut miles in position and 1 fps in velocity. The factors involved in the establishment of these requirements are described in Ref. 1 for an orbital-rendezvous mission. The major consideration here is that the ferry be transferred from parking orbit to a point sufficiently close to the satellite so that its radar seeker will be able to lock on to the satellite. For the case of a soft landing on the moon, it is important that the touchdown velocity be accurately controlled both in magnitude and direction in order to prevent tumbling and damage to the landing vehicle. For a representative three-legged landing vehicle, it is desirable that the vertical component of landing velocity be at least twice as large as the horizontal component to insure satisfactory stability.[2] If an inertial guidance system is used for determination of landing velocity, then it is important that the vehicle's velocity be accurately determined in the lunar parking orbit, prior to the final descent.

The most reliable method for sensing the horizon discontinuity of a planet such as the earth or moon is by measurement of the sharp gradient of infrared radiation between the edge of the body and outer space.[3] For the best performance it is desirable that the horizon sensor respond only to emitted thermal radiation from the planet, ignoring the reflected solar radiation.[4]

There are many different methods that have been developed and proposed for determining the horizon discontinuity of a planet.[3–8] One type of scanning system, which has been successfully used in experimental Atlas and Thor nose cones, test flown in a Jupiter missile, and is being used in the Project Mercury space capsule is shown in Fig. 1. The field of view of the detector (2° × 8°) is deflected by a scanning prism at an angle of 55° from the normal. A drive motor rotates the scanning prism at 30 rps, forming a 110° conical scan pattern as shown in Fig. 1. The difference in radiance level between the planet and outer space detected by the sensor causes a 30 cps rectangular wave to be generated. By comparing the phase of this rectangular wave with an internally generated reference signal, it is possible to determine the orientation of the pitch or roll axis of the vehicle with respect to the local vertical. A pair of these horizon sensors, oriented at right angles to each other as in Fig. 1, determine the orientation of the vehicle relative to the vertical.

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